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CORRELATIONS OF CONCENTRATION, TEMPERATURE AND VELOCITY PROFILES IN COMPRESSIBLE TURBULENT BOUNDARY LAYERS WITH FOREIGN GAS INJECTION

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Abstract—In a recent experimental investigation measurements have been made of the velocity, temperature and concentration profiles along a porous flat plate with carbon dioxide injected at the surface. In this paper the relationships between these profiles are considered. It is found that the local concentration and velocity can be correlated by using an assumption of a constant turbulent Schmidt number through the layer for given conditions. This constant Schmidt number is significantly less than unity and varies with skin friction and injection rate. The local temperature can be related to the local velocity and concentration by an equation which is an extended form of previous relationships for single-component boundary layers.

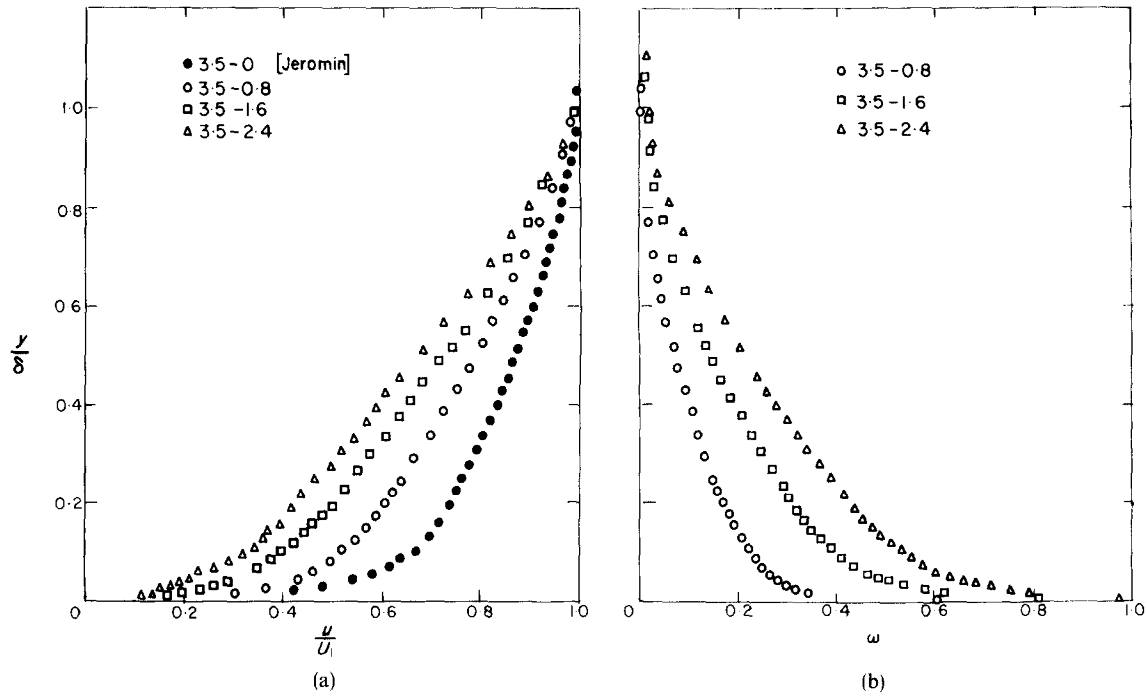
NOMENCLATURE

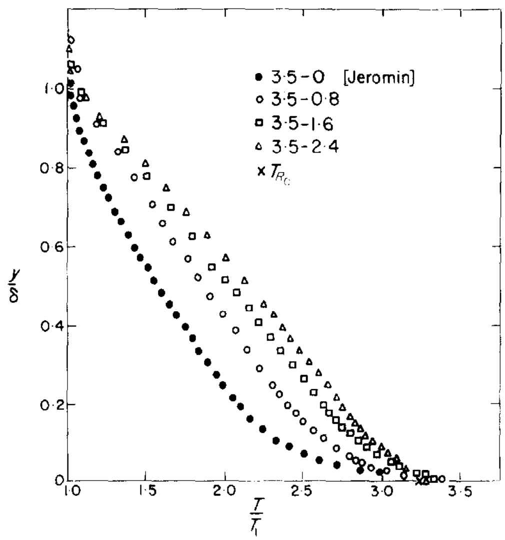
c_f, skin-friction coefficient;
c_p, specific heat at constant pressure;
c_{pa}, specific heat at constant pressure for air;
c_{pc}, specific heat at constant pressure for injected gas;
F, injection parameter $(= \rho_w v_w/\rho_1 U_1)$;
h, specific enthalpy;
h_c, specific enthalpy of injected gas;
M, Mach number;
Pr_t, turbulent Prandtl number;
q_w, conduction heat-transfer rate per unit area at wall;
r, recovery factor;
Sc_t, turbulent Schmidt number;
T, temperature;
u, U, velocity in the x direction;
u^*, $= u/U_1$;
v, velocity normal to the surface;
x, distance along the surface in the stream direction;
y, distance normal to the surface;
δ, boundary layer thickness $(u/U_1 = 0{\cdot}995)$;
ε_d, ε_v, eddy coefficients of diffusion and viscosity;
τ_w, shear stress at wall;
ρ, density;
ω, mass fraction of injected gas;
θ, momentum thickness.

Subscripts
r, recovery conditions;
w, wall conditions;
0, zero injection conditions;
1, conditions at edge of the boundary layer.

1. INTRODUCTION

In recent years there has been considerable interest in fluid injection into a turbulent boundary layer as a method of cooling a body exposed to a hot gas stream. However, in spite of this wide interest there have been relatively





(c)

FIG. 1. (a) Velocity profiles at $M = 3{\cdot}5$. (b) Concentration profiles at $M = 3{\cdot}5$. (c) Temperature profiles at $M = 3{\cdot}5$.

few complete investigations of the boundary layer development with foreign gas injection. To remedy this deficiency the first author has recently completed an experimental investigation (Dunbar [1]) of the velocity, temperature and concentration profiles in a compressible turbulent boundary layer along a porous flat plate with carbon dioxide injected through the plate. The measurements were made at Mach numbers from 0·55 to 3·6 and form a natural continuation of the tests with air injection made by Jeromin [2] and Squire [3]. Full details of the measurements are given in [1]. In this paper the relationships between the concentration, temperature and velocity profiles are considered.

In general these correlations are based on Couette-flow analysis with simple assumptions for the variations of the diffusion parameters. However, before discussing these correlations in detail it is helpful to consider the overall results as shown in Fig. 1 where typical measured profiles are presented at one Mach number and one measuring station†. In the figures the main effects of injection are clearly seen, as are the many qualitative similarities between the three sets of profiles. For example the changes in all the profiles are always less as the injection rate is increased from 0·0016 to 0·0024, than are the changes as the injection rate is increased from 0 to 0·0008. Of particular interest is the strong similarity between the shapes of the velocity and concentration profiles. In all cases the measured quantities change very rapidly in the region very close to the wall ($y/\delta < 0{\cdot}02$). Thus although measurements could be made to within 0·08 mm of the wall, these measurements are not representative of wall conditions. This causes particular difficulties in the case of the concentration profiles since it was impossible to make independent measurements of the wall concentration. (The wall temperature could be measured by thermocouples set into the surface, and it may be assumed that $u = 0$ at the wall.) Thus before any correlations can be considered it is necessary to estimate the wall concentration. This estimation is considered in an appendix, together with a brief account of the experiments.

† The notation used in this paper to define the profiles gives the nominal Mach number as the first number in the profile label, and the value of the injection parameter $(\rho_w v_w/\rho_1 U_1)$ multiplied by 10^3 as the second number.

2. CORRELATION OF THE CONCENTRATION AND VELOCITY PROFILES

2.1 *The theoretical correlation*

The correlation used in this paper follows directly from the analysis of Rubesin and Pappas [4], and is given by

$$1 - \omega = \left[\frac{\dfrac{2F}{c_f}\dfrac{u}{U_1} + 1}{\dfrac{2F}{c_f} + 1}\right]^{Sc_t} \tag{1}$$

This relationship is obtained by making the following assumptions:

(i) the Couette-flow approximations hold throughout the boundary layer,

(ii) $\omega = 0$ at $u = U_1$,

(iii) the turbulent Schmidt number is constant.

From equation (1) the wall concentration is given by

$$\omega_w = 1 - \left[\frac{1}{\dfrac{2F}{c_f} + 1}\right]^{Sc_t} \tag{2}$$

For the case of a Schmidt number of one the relationship reduces to

$$\frac{\omega}{\omega_w} = 1 - \frac{u}{U_1}. \tag{3}$$

It should be noted that Rubesin and Pappas do not use equation (2), but match equation (1) to a concentration profile in the laminar sub-layer, whereas Spalding *et al.* [5] use equation (3) throughout the layer. A similar method to that used by Rubesin and Pappas was also used by Ness [6], except that he assumed that the turbulent Schmidt number was constant at a

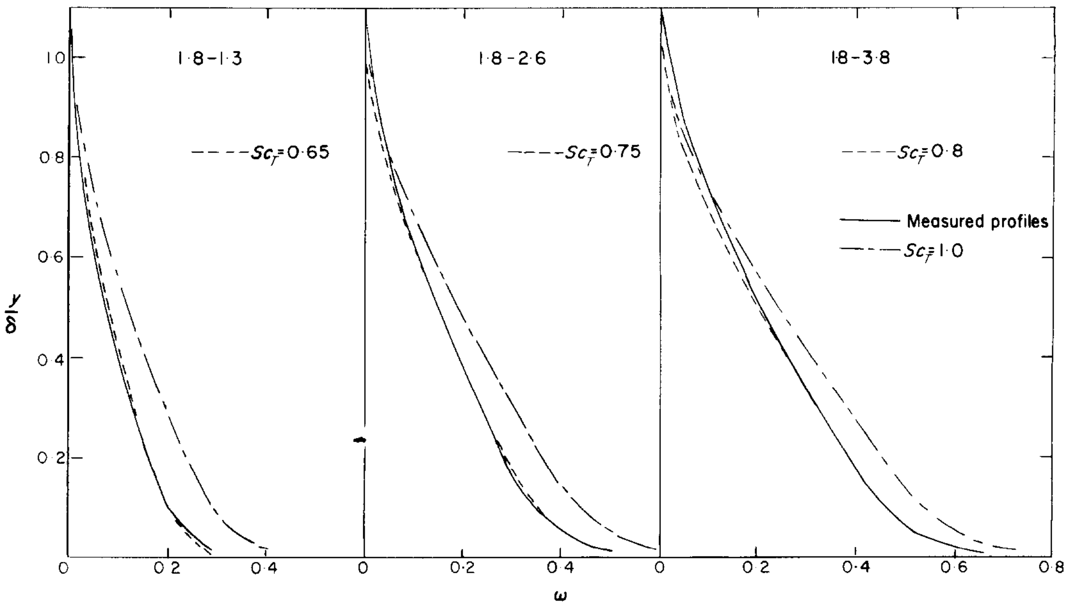


FIG. 2. Comparison of measured and deduced concentration profiles at $M = 1 \cdot 8$.

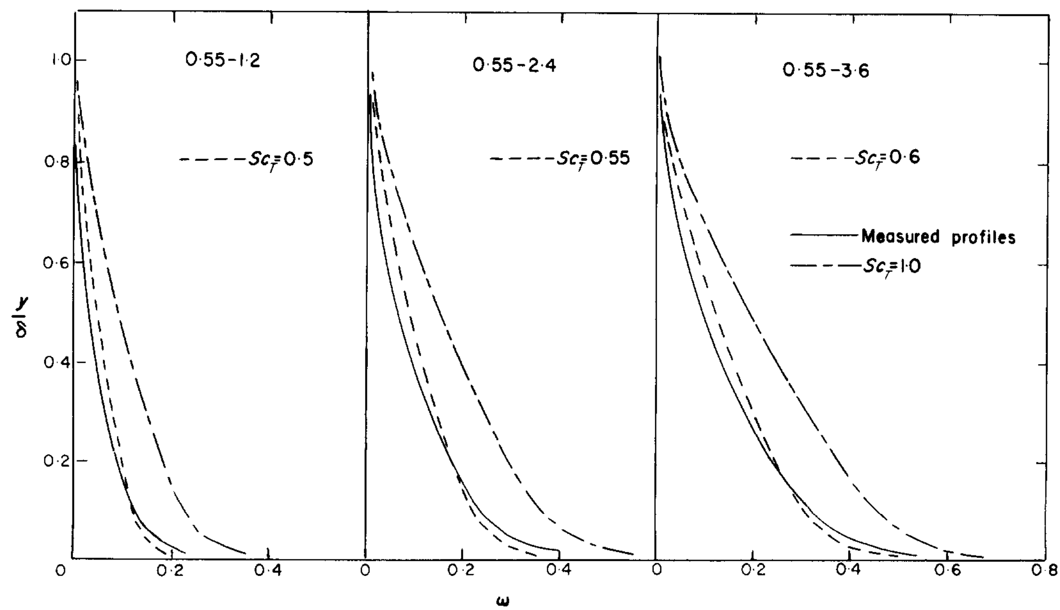


FIG. 3. Comparison of measured and deduced concentration profiles at $M = 0 \cdot 55$.

value equal to that at the edge of the sublayer, whereas Rubesin and Pappas assume a turbulent Schmidt number of one. Therefore, a comparison of equations (1)–(3) with the present experimental data will provide evidence for the validity, or otherwise of previous theoretical assumptions.

2.2 *Comparison of the theory and experiment*

The theoretical correlations of the last section were compared with experiment by a comparison of the measured concentration profile with those deduced from the measured velocity profiles using equation (1). In this comparison the turbulent Schmidt number was treated as a free parameter which could be varied to give the best overall agreement between the measured and the deduced concentration profile.

Results of this comparison are shown in Figs. 2 and 3 where results for $M = 0{\cdot}55$ and $1{\cdot}8$ are plotted as ω against y/δ. These figures also include the deduced profiles for a turbulent Schmidt number of one, i.e. equation (3). For all the profiles, by choosing a suitable value for Sc_t, reasonable quantitative agreement between the two profiles is obtained, the maximum difference being about 3 per cent of carbon dioxide. In general the worse agreement occurred at $M = 0{\cdot}55$ and the best at $M = 1{\cdot}8$ and $2{\cdot}5$.

The effect of variations in the value chosen for Sc_t can be seen from those figures where the curves corresponding to a turbulent Schmidt number of unity are included. It is apparent that the deduced concentration profile is sensitive to the chosen value of this parameter, the relative change in the concentration at a point being of the same order, and having the same sign, as the relative change in Sc_t. Thus the value of Sc_t which gives the best agreement with the measurements can be determined to within 0·03. The value of Sc_t obtained also depends on the value assumed for $2F/c_f$. In general it was found that the likely uncertainty in c_f added an additional uncertainty of $\pm 0{\cdot}02$ to the value of Sc_t. It can also be seen from these figures that the assumption of a turbulent Schmidt number of unity does not give good correlation for the present results, the difference between the deduced and measured concentration being as great as 50 per cent in some profiles.

The values of the constant turbulent Schmidt number which were found to provide the best correlation are shown in Fig. 4 plotted against

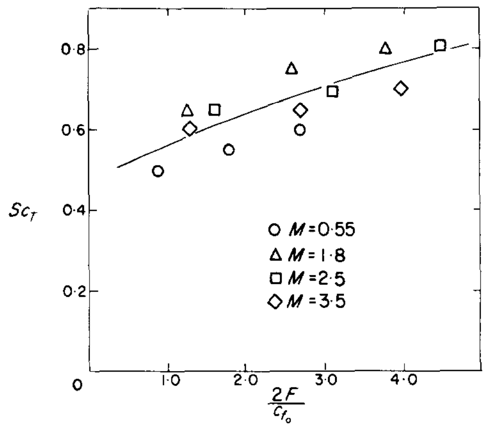


FIG. 4. Variation of the constant turbulent Schmidt number with injection rate.

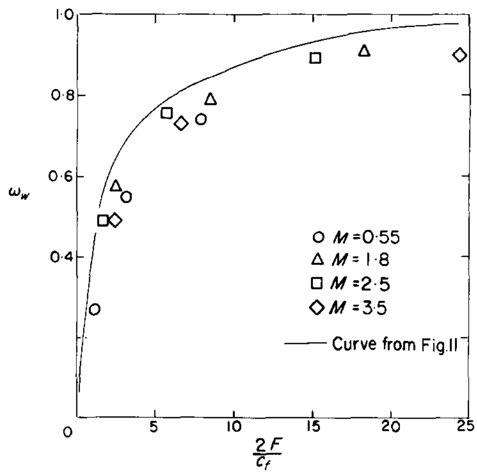


FIG. 5. Variation of the calculated surface concentration with injection rate.

the injection parameter $2F/c_{f0}$. The observed values lie between 0·5 and 0·8, those corresponding to $M = 0{\cdot}55$ being generally lower than those found in the supersonic boundary layers.

For each Mach number it is, of course, possible to draw a line showing the variation of Sc_t with $2F/c_{f_0}$. However, the behaviour, with varying Mach number, of these variations is not apparently systematic and this approach is not very informative, especially since the possible error of 0·05 in Sc_t measurements is of the same order as the differences between the results for the various Mach numbers. However, one consistent feature of the variation at each Mach number is that the turbulent Schmidt number increases with increasing injection rate. Finally, from Figs. 2 and 3 it can be seen that in the region very close to the wall the difference between the deduced and measured concentration profiles tends to increase. These differences are clearly seen in Fig. 5 where the surface concentration from equation (2) (using the relevant values of Sc_t from Fig. 4) are shown together with the estimated surface concentrations from the appendix (Fig. 11). It can be seen that, even for profiles where the present correlation is very good, the values of the surface concentration from equation (2) are up to 10 per cent less than the experimental values. Thus the present correlation does not hold in the region very close to the wall ($y/\delta < 0{\cdot}01$, say).

3. CORRELATION OF TEMPERATURE WITH CONCENTRATION AND VELOCITY

For high-speed boundary layers on solid walls with negligible heat transfer, a first approximation to the temperature distribution can be obtained from the assumption of constant stagnation temperature throughout the boundary layer. This approach, however, takes no account of the internal heat transfer in the boundary layer which causes the adiabatic surface temperature to be lower than the free-stream stagnation temperature. With heat transfer or fluid injection, the surface temperature can differ considerably from the free-stream stagnation value and so this approach has obvious shortcomings. However, it has been shown by Crocco [7] and van Driest [8] that the assumption of constant stagnation temperature corresponds to one of unity turbulent Prandtl number and, therefore, the analysis could be extended to include solid wall boundary layers with heat transfer. Under these conditions the temperature relation becomes

$$\frac{T}{T_1} = \frac{T_w}{T_1} - \frac{T_w - T_1}{T_1} u^* + \frac{\gamma - 1}{2} M_1^2 u^*(1 - u^*). \quad (4)$$

From his theoretical analysis considering constant turbulent Prandtl numbers from 0·6 to 1·4, Spence [9] found that a good approximation to his computed results was given by

$$\frac{T}{T_1} = \frac{T_w}{T_1} + \frac{T_r - T_w}{T_1} u^* - \frac{T_r - T_1}{T_1} u^{*2} \quad (5)$$

where the subscript r refers to adiabatic wall (or recovery) conditions. This reduces to equation (8) on the assumption of unit turbulent Prandtl number. Danberg *et al.* [10] and Jeromin [2] found that equation (9) gave good agreement with their experimental results for air injection into turbulent boundary layers at Mach numbers from 2·5 to 6·7.

The aim of the present section is to find a similar relationship for the temperature in the boundary layer with foreign gas injection.

3.1 *A theory for the enthalpy–velocity relation*

The initial part of the theory again follows the analysis of Rubesin and Pappas [4] using the Couette-flow approximations. By integrating the energy equation with boundary conditions at the surface, and using the momentum equation they obtain the equation ([4], equation (50))

$$\frac{\rho_w v_w (h - h_{cw} + u^2/2) - q_w}{\rho_w v_w u + \tau_w} = \frac{1}{Pr_t}\frac{dh}{du} + u. \quad (6)$$

Integration of this equation with $h = h_w$ at $u = 0$ and $Pr_t = 1$ gives

$$h - h_w + u^2/2 = \frac{\rho_w v_w u}{\tau_w}(h_w - h_{cw}) - \frac{q_w}{\tau_w} u. \quad (7)$$

If it is now assumed that $h = h_1$ at $u = U_1$ equation (7) gives

$$-\frac{q_w}{\tau_w} = \frac{h_1 - h_w}{U_1} + \frac{U_1}{2} - \frac{\rho_w v_w (h_w - h_{cw})}{\tau_w} \qquad (8)$$

so that q_w/τ_w may be eliminated from equation (7). Finally the results may be generalized to the case of non-unit Prandtl number by assuming that in the absence of heat transfer the wall enthalpy is equal to the recovery enthalpy, rather than to the free stream stagnation enthalpy. Thus from equation (8) $q_w = 0$ when

$$\frac{h_1 - h_r}{U_1} + \frac{U_1}{2} - \frac{\rho_w v_w (h_r - h_{cr})}{\tau_w} = 0 \qquad (9)$$

provided τ_w is unaffected by this change. Substitution of (8) and (9) into equation (7) then gives

$$h = h_w + (h_r - h_w)(u/U_1) - (h_r - h_1)(u/U_1)^2 - (2F/c_f)(h_r - h_{cr})\,[(u/U_1)^2 - (u/U_1)]. \qquad (10)$$

This is the relationship required. It can be converted to a temperature relationship by making the perfect gas assumption that

$$h = c_p T, \text{ where } c_p = (1 - \omega)\, c_{pa} + \omega c_{pc}.$$

It can be seen that for the case of solid surfaces (i.e. $F = 0$) the relationship reduces to that proposed by Spence. Also for air injection the term $(h_r - h_{cr})$ is zero under all conditions and so the relationship again reduces to equation (5). This is in agreement with the observations of Danberg and Jeromin. Thus equation (10) gives a good description of previous investigations of single-component turbulent boundary layers.

Before equation (10) can be compared with the present experimental results it is necessary to be able to specify the adiabatic surface conditions. For the present investigation it is assumed that the heat transfer is sufficiently small that changes in the skin-friction and surface concentration are negligible. Therefore the only surface condition that is required is the recovery temperature, or the recovery factor.

Experimental information about the variation of the recovery factor with injection of air, or other gases, is extremely sparse and has not been systematized to any great extent. No data could be found which had been obtained with carbon dioxide injection. However, the results of Pappas and Okuno [11] show that for the injection of Freon-12 (a heavier gas than carbon dioxide having a molecular weight of about 120) the variation of recovery factor with injection rate is, within experimental accuracy, the same as that for air injection. Because the difference between the effects caused by different injected gases are, to a large extent, dependent on the molecular weight of the gas, it was assumed that the variation of the recovery factor with carbon dioxide injection was also negligibly different from that with air injection.

Even for air injection, experimental measurements have not defined a systematic variation of recovery factor which is universally applicable. However, Spalding *et al.* [5] found that the measurements of Bartle and Leadon [12] and Leadon and Scott [13] at Mach numbers from 2·0 to 3·2, could be described, with fair accuracy, by the empirical relation

$$\frac{r}{r_0} = \left(1 + 0{\cdot}83\,\frac{2F}{c_f}\right)^{-0.04}. \qquad (11)$$

For want of more reliable information, equation (11) was used to calculate the variation of the recovery factor with carbon dioxide injection rate. The zero injection recovery factor r_0 was assumed to have the generally accepted value of 0·89

3.2 *A comparison of the theory with experiment*

Using the results of section 3.1 equation (10) can be used to derive a temperature profile from the measured velocity and concentration profiles. This calculation was performed for the supersonic profiles but not for the subsonic cases because, as was explained in [1], the

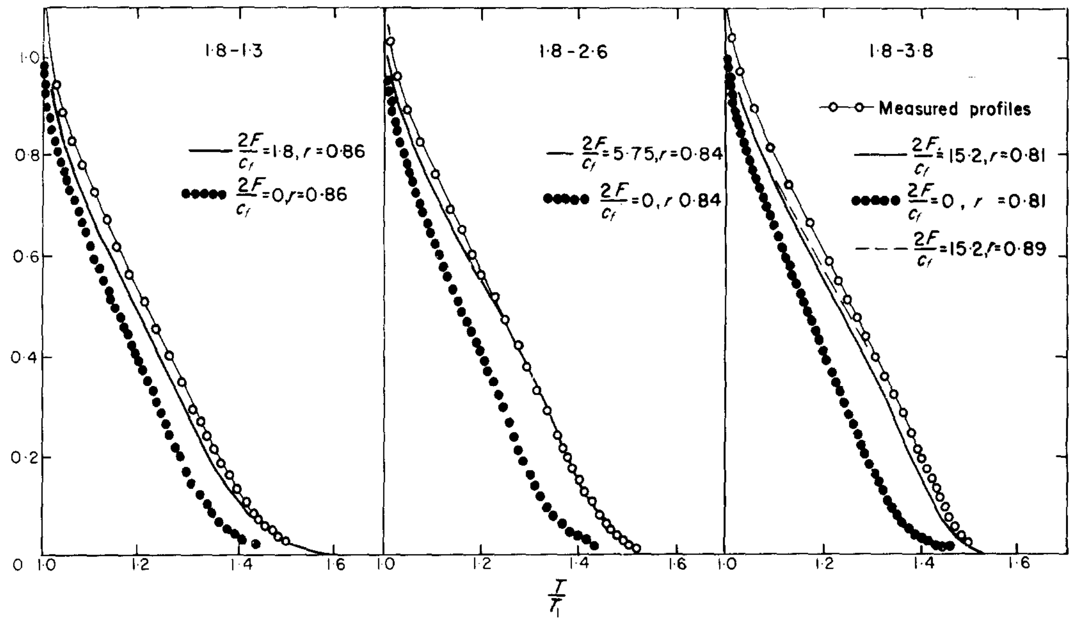


FIG. 6. Comparison of measured and deduced temperatures at $M = 1{\cdot}8$.

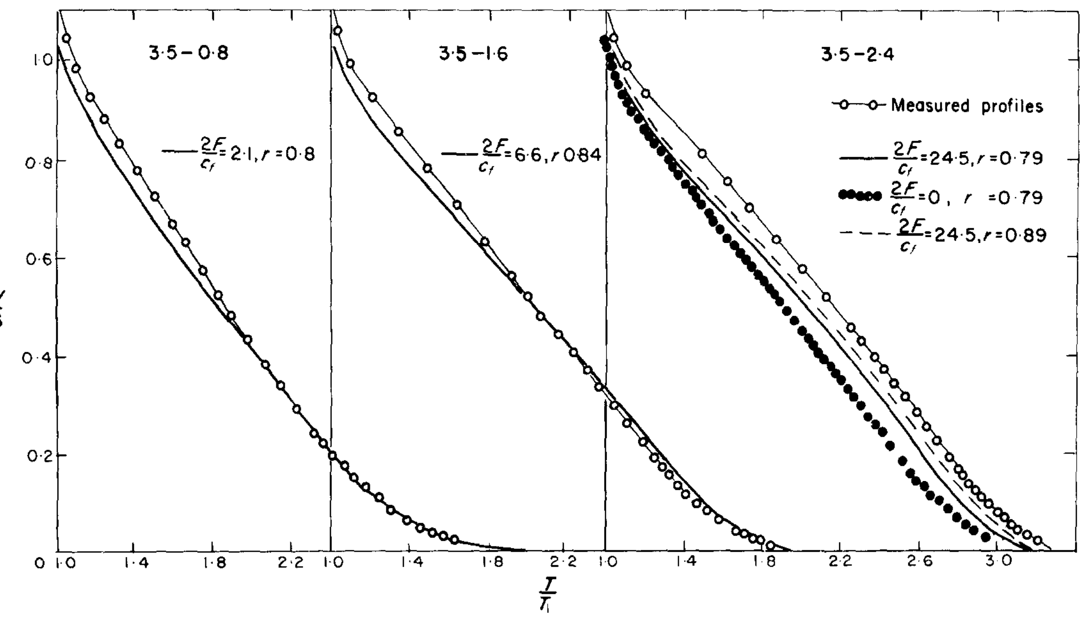


FIG. 7. Comparison of measured and deduced temperatures at $M = 3{\cdot}5$.

detailed low speed temperature profiles are suspect.

The results of the calculations for the profiles at $M = 1{\cdot}8$ and $3{\cdot}5$ are shown in Figs. 6 and 7, plotted in the form of T/T_1 against y/δ.

For the highest injection rate at each Mach number two further calculations were performed. Firstly, a temperature profile was derived corresponding to the value of r given by equation (11) and a value of $(2F/c_f)$ of zero. These profiles give some indication of the size of the last term in equation (10), also they show the effects of possible inaccuracies in the parameter $2F/c_f$. This calculation was also made for all the profiles at $M = 1{\cdot}8$ (Fig. 6). Secondly, temperature profiles were derived corresponding to the measured value of $2F/c_f$ but with a recovery factor of 0·89. These profiles give an indication of the effect of inaccuracies in the assumed value of the recovery factor.

It can be seen that, in general, the agreement between the measured temperature profiles and those deduced from equation (10), using the nominal values of $2F/c_f$ and r, is good, especially in the region $0 < y/\delta < 0{\cdot}5$. The profiles for 1·8–2·6, and 3·5–0·8/1·6 are of special interest, because, for these profiles, the agreement in this region is extremely good. However, in the region $y/\delta > 0{\cdot}5$ the difference between these correlated and measured profiles increases to as much as 3 per cent of $(T_w - T_1)$. In this region the correlated profile always predicts a lower temperature than that which was measured. This lower temperature arises from the boundary condition $h = h_1$ at $u = U_1$ imposed on equation (10), whereas in fact the temperature boundary layer is always slightly thicker than the velocity boundary layer so that $h > h_1$ at $u = U_1$.

The largest differences between the correlated and measured temperature profiles occur at the highest blowing rates at $M = 2{\cdot}5$ and $3{\cdot}5$ where the differences are of the order of 7 per cent of $(T_w - T_1)$. At these high blowing rates the shape of the correlated temperature profiles is very sensitive to the assumed values of c_f, recovery factor and wall concentration, and it was found that the possible errors in these quantities could account for most of the differences between the correlated and measured profiles.

4. CALCULATION OF THE TURBULENT SCHMIDT NUMBER

In addition to the correlations discussed above an attempt was made to use the present results to find the actual values of the turbulent Schmidt number in the injected boundary layer.

In order to find the turbulent Schmidt number at a point in the boundary layer, it is necessary to determine the eddy viscosity, ε_v, and the quantity $\rho\varepsilon_d$ which appears in the diffusion equation. The eddy viscosity can be found by dividing the local shear stress by the velocity grandient du/dy. To determine $\rho\varepsilon_d$ it is necessary to find the diffusion mass flow of injected gas per unit area in a direction normal to the surface at any point in the boundary layer. Dividing this mass flow rate by the local concentration gradient, $d\omega/dy$, gives the quantity $\rho\varepsilon_d$. The injected gas diffusion rate and the shear stress could be found by integrating the equations of motion and using consecutive profiles to give the derivatives in the flow direction. However, the irregularities of the profile developments caused by the small pressure variations along the plate, together with the general experimental scatter makes this method impracticable. The difficulty was resolved by using the fact that at the front of the measuring region consecutive profiles were similar (as shown in [3]) when plotted against y/θ and thus derivatives in the x-direction could be found in terms of $d\theta/dx$. Details of the actual process are given in [1].

From a consideration of the various sources of error which occur in this process it appears that the calculated values of the Schmidt number may be considered reasonably accurate over the inner 50 per cent of the boundary layers at supersonic speeds, but only over the inner 25 per cent of the boundary layer at $M = 0{\cdot}55$. The

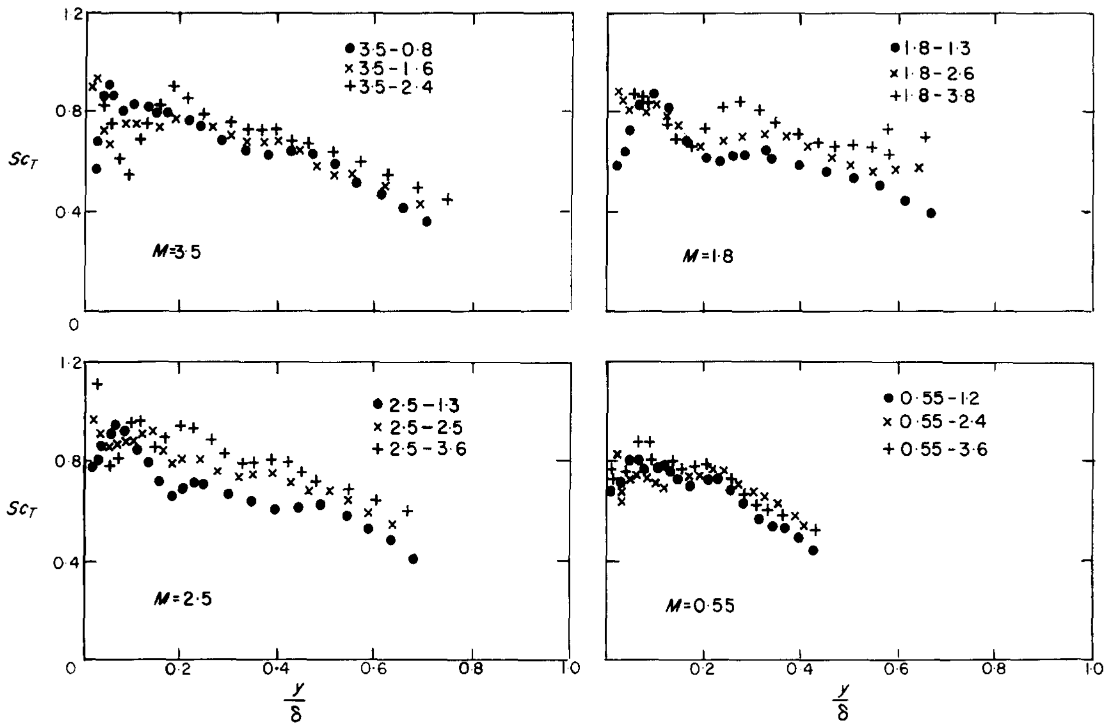


FIG. 8. Schmidt number profiles.

deduced values of the Schmidt number in these ranges are plotted in Fig. 8.

The behaviour of the turbulent Schmidt number in the inner region, as shown by Fig. 8, is similar for all the boundary layers. Near the surface Sc_t has a value of the order of 0·8 and it decreases slightly with increasing y/δ, changing by the order of 0·1 in the distance from near the surface to $y/\delta = 0{\cdot}4$. It can be seen that, at each Mach number, the level of the turbulent Schmidt number distribution tends to increase with increasing injection rate. A good indication of this level is the value of Sc_t at $y/\delta = 0{\cdot}2$. In Fig. 9 this quantity is plotted against $2F/c_{f_0}$ for all the measured profiles and it can be seen that the points lie in a narrow band whose mean Sc_t value increases with increasing $2F/c_{f_0}$. This variation defines, to fair accuracy, the magnitude of the turbulent Schmidt number in the part of the boundary layer near the surface. As stated above the magnitude of Sc_t in the outer part cannot be defined accurately from these experimental results.

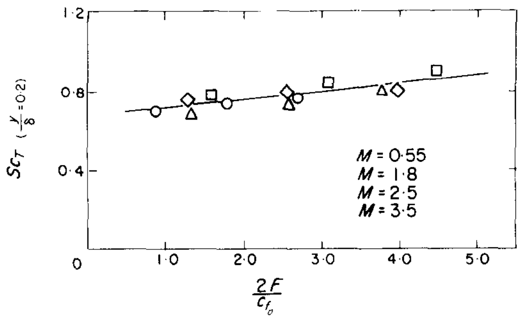


FIG. 9. Variation of Schmidt number at $y/\delta = 0{\cdot}2$ with injection rate.

It is important to note that these measurements do not give any indication of the behaviour of Sc_t in the region very close to the wall where the flow is more viscous in nature.

Because the Schmidt number which has been determined is, effectively, the total Schmidt number, its value at the wall should be that of the laminar Schmidt number corresponding to wall conditions. For carbon dioxide–air mixtures the laminar Schmidt number varies from about 0·5 for 100 per cent carbon dioxide to about 0·95 for 100 per cent air. Therefore, since the wall concentration of carbon dioxide in these experiments is generally greater than 50 per cent, the Schmidt number must vary rapidly, close to the wall, from the surface value, of the order of 0·6, to that measured in the inner turbulent region, of the order of 0·8.

5. CONCLUSIONS

The results of the analysis of this paper can be summarised as follows:

(i) The assumptions of Couette-type flow and constant turbulent Schmidt number for each Mach number and injection rate can be used to provide an analytic relation between the local concentration and velocity throughout the boundary layer. This relation gives good correlation of the present experimental results. The behaviour of this constant Schmidt number with varying Mach number and injection rate has been partially systematised.

(ii) The assumption of a constant turbulent Schmidt number of unity does not give good agreement with the present experimental results.

(iii) Integration of the Couette-flow equations of motion gives a relation between the local enthalpy and velocity which is in agreement with previous relations which were applicable to single-component boundary layers.

(iv) Making perfect gas assumptions, this relation gives a good correlation of the results of the present experimental programme.

(v) Values of the local turbulent Schmidt number deduced from the measurements indicate that it has a value of the order of 0·8 in the inner region and tends to decrease towards the edge of the boundary layer. The magnitude of the turbulent Schmidt number in the inner region increases with injection rate.

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APPENDIX

(a) *Brief details of the test programme*

All the measurements were made in the supersonic wind tunnel in the Cambridge University Engineering Department. In normal use this is an intermittent blowdown tunnel with a pair of symmetrical nozzle blocks and a working section of 114 mm × 170 mm. However, in order to fit the injection equipment it was necessary to modify the tunnel and a flat porous test plate was placed along

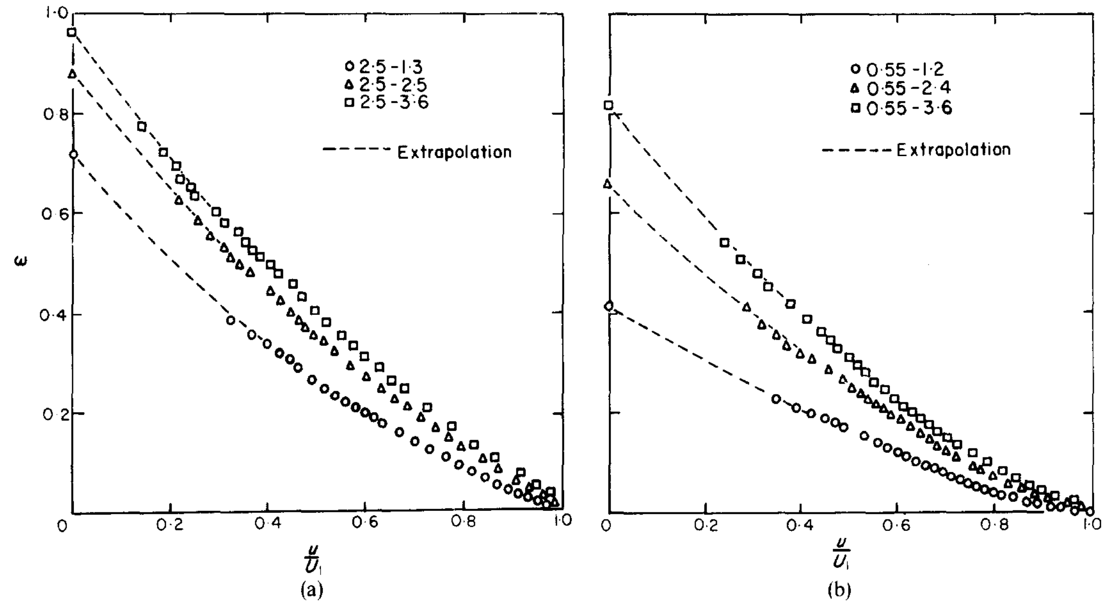

FIG. 10. (a) Variation of concentration with velocity at $M = 2{\cdot}5$. (b) Variation of concentration with velocity at $M = 0{\cdot}55$.

the centre line of the tunnel. The space below the test plate was used as a plenum chamber for the injected air. This arrangement meant that only one nozzle block was required to generate the flow and that the test section was reduced to 114 mm × 85 mm. However, the tunnel running time was doubled to a maximum of about 150 s.

For tests with carbon dioxide injection traverses were made through the boundary layer using flattened pitot tubes and conical temperature probes. In addition a sampling probe was used to obtain concentration profiles. These measurements were used to obtain velocity, temperature

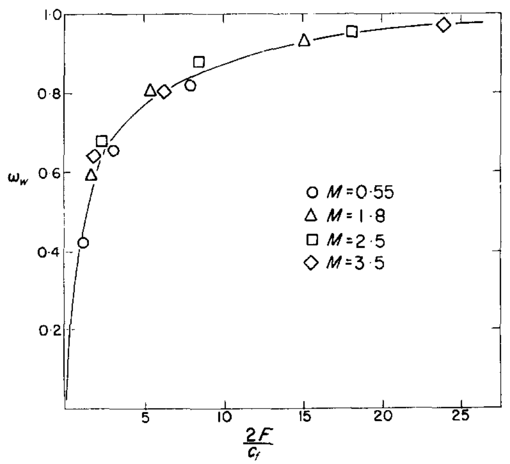

FIG. 11. Variation of surface concentration with injection rate.

Table 1. Experimental conditions†

Mach number	Injection rate F $\times 10^3$	R_θ at front station $\times 10^{-4}$	c_f $\times 10^3$
0·55	0	1·7	2·7
0·55	1·2	2·0	2·1
0·55	2·4	2·3	1·5
0·55	3·6	2·6	0·9
1·8	0	0·9	2·0
1·8	1·35	1·4	1·5
1·8	2·6	1·9	0·9
1·8	3·8	2·4	0·5
2·5	0	1·0	1·6
2·5	1·3	1·7	1·0
2·5	2·5	2·3	0·7
2·5	3·6	3·0	0·4
3·5	0	1·5	1·2
3·5	0·8	2·1	0·8
3·5	1·6	2·9	0·5
3·5	2·4	3·6	0·2

† The accuracy of the quoted values of c_f is $\pm 0{\cdot}0001$ at low injection rates, rising to $\pm 0{\cdot}0002$ at the higher rates.

each Mach number and injection rate three profiles were measured along the plate. The first profile was measured at 200 mm from the leading edge of the porous plate and the other two profiles at 63 mm intervals downstream. The test conditions are summarised in the Table 1. The skin-friction coefficients quoted in this table were obtained from the measured momentum growth along the plate.

(b) *Determination of the surface concentration*

From a comparison of the experimental velocity and concentration profiles it is apparent that the region of the boundary layer in which the concentration varies rapidly with distance from the wall corresponds closely to that in which the velocity varies rapidly. It would therefore appear possible that, by plotting the variation of concentration with velocity, a curve might be obtained with a smaller rate of change of slope near the surface. This curve could then be extrapolated in order to estimate the conditions at, and near, the surface.

Sample experimental results have been plotted in Fig. 10 as ω against u^*. It can be seen that the experimental points form curves which vary smoothly throughout the boundary layer, especially in the region corresponding to measurements near the wall.

The surface concentrations determined by the method of extrapolation are used throughout this paper. They are shown in Fig. 11, where ω_w is plotted against $2F/c_f$. All the points lie, within experimental error, on a single line which is shown in the figure. This variation of ω_w with $2F/c_f$ is qualitatively similar to that found analytically by Rubesin and Pappas [4] for isothermal boundary layers with hydrogen and helium injection. It can also be seen that the surface concentration at low injection rates rises very rapidly with increasing $2F/c_f$ and, at high values of $2F/c_f$, ω_w tends to 1·0.

CORRÉLATIONS ENTRE LES PROFILS DE CONCENTRATION, DE TEMPÉATURE ET DE VITESSE DANS DES COUCHES LIMITES TURBULENTES COMPRESSIBLES AVEC INJECTION D'UN GAZ EXTÉRIEUR

Résumé—Au cours d'une recherche expérimentale récente, on a déterminé des profils de vitesse, de température et de concentration le long d'une plaque plate poreuse, avec injection pariétale de gaz carbonique. On considère ici, les relations existant entre ces profils. On trouve que la vitesse et la concentration locales peuvent être reliées en utilisant l'hypothèse d'un nombre de Schmidt turbulent constant à travers la couche limite dans des conditions données. Ce nombre de Schmidt constant est de façon significative inférieur à l'unité, et varie avec le frottement superficiel et le taux d'injection. La température locale peut être reliée à la vitesse et la concentration locales à l'aide d'une équation qui est une extension de relations déjà établies pour des couches limites à un seul composant.

BEZIEHUNGEN ZWISCHEN KONZENTRATIONS-, TEMPERATUR- UND GESCHWINDIGKEITSPROFILEN IN KOMPRESSIBLEN TURBULENTEN GRENZSCHICHTEN MIT FREMDGASAUSBLASUNG

Zusammenfassung—In einer kürzlich durchgeführten experimentellen Untersuchung sind Messungen der Geschwindigkeits-, Temperatur- und Konzentrationsprofile entlang einer porösen ebenen Platte mit Kohlendioxid gemacht worden, das an der Oberfläche ausgeblasen wurde. In dieser Veröffentlichung werden Verwandtschaften zwischen diesen Profilen betrachtet. Es wurde gefunden, dass örtliche Konzentration und Geschwindigkeit in Beziehung gesetzt werden können, wenn man für die gegebenen Bedingungen eine konstante Schmidt-Zahl annimmt. Diese konstante Schmidt-Zahl ist erheblich geringer als eins und ändert sich mit der Oberflächenreibung und der Ausblasemenge. Die örtliche Temperatur kann mit der örtlichen Geschwindigkeit und Konzentration durch eine Gleichung in Verbindung gebracht werden, die man als erweiterte Form früherer Beziehungen für Einkomponenten-Grenzschichten betrachten kann.

КОРРЕЛЯЦИИ РАСПРЕДЕЛЕНИЙ КОНЦЕНТРАЦИИ, ТЕМПЕРАТУРЫ И СКОРОСТИ В СЖИМАЕМЫХ ТУРБУЛЕНТНЫХ ПОГРАНИЧНЫХ СЛОЯХ ПРИ ВДУВЕ ИНОРОДНОГО ГАЗА

Аннотация—Проводились экспериментальные измерения распределений скорости, температуры и концентрации вдоль плоской пористой пластины при вдуве двуокиси углерода на поверхность. В данной работе рассмотрены соотношения между этими распределениями. Установлено, что локальные концентрации и скорости можно обоб-

щить в предположении постоянного турбулентного числа Шмидта в слое для данных условий. Это постоянное число Шмидта значительно больше единицы и изменялось в зависимости от поверхностного трения и скорости вдува. Локальную температуру можно связать с локальной скоростью и концентрацией уравнением, полученным в результате обобщения полученных ранее соотношений для однокомпонентных пограничных слоев.